The exceptional Jordan algebra and the matrix string

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ABSTRACT

A new matrix model is described, based on the exceptional Jordan algebra, $J^3_{\mathcal{O}}$. The action is cubic, as in matrix Chern-Simons theory. We describe a compactification that, we argue, reproduces, at the one loop level, an octonionic compactification of the matrix string theory in which $SO(8)$ is broken to $G2$. There are 27 matrix degrees of freedom, which under $Spin(8)$ transform as the vector, spinor and conjugate spinor, plus three singlets, which represent the two longitudinal coordinates plus an eleventh coordinate. Supersymmetry appears to be related to triality of the representations of $Spin(8)$.

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Contents

1 Introduction

This letter proposes a new answer to the question of what is the background independent formulation of string theory. This proposal is based on the use the exceptional Jordan algebra, whose structure is shown to code the degrees of freedom of string theory. The dynamics of the theory is expressed in a purely algebraic framework, which is a background independent matrix model, whose action makes no reference to a metric or classical manifold. Space and time, as well as the string and branelike excitations, arise by expanding the theory around some of its classical solutions.

While this model has a unique algebraic structure, it falls into a general class of models previously studied called the cubic matrix models[1, 2, 3], which are supersymmetric extensions of matrix Chern-Simons theory. These models are of interest because they appear to provide a unification of string theory with a class of background independent theories. These latter are extensions of loop quantum gravity[4], which were previously studied as a possible background independent framework for $\mathcal{M}$ theory[5, 6, 7].

Just as string theory appears to be the unique framework for constructing quantum gravity in the background dependent regime, loop quantum gravity[4] is so far the only well worked out framework we have for describing quantum gravity in a background independent fashion. The dynamics can be expressed in either a hamiltonian[4] or path integral formalism[8, 9], when the latter is used close relations to topological field theory and dynamical triangulations are revealed. The framework is quite general, and accommodates not only quantum general relativity[4], but supergravity[10], branes[11] and higher dimensional theories[12, 13]. The framework also provides a natural formulation of the holographic principle which makes sense in cosmological and background independent contexts[14]. Since the real quantum theory of gravity must be formulated in a background independent manner, yet have a consistent description at the background dependent level, it is reasonable to seek the theory by means of a unification of loop quantum gravity and string theory. The search for this unification[5, 6, 7, 15] led to the discovery of the cubic matrix models, which appear to have both background dependent phases that reproduce the known matrix models for string theory[1, 3] and Chern-Simons-like phases that lead to a version of loop quantum gravity with the symmetries appropriate to string theory[2].

The next question facing this approach is whether there may be within it a single theory based on a unique mathematical structure which could serve as a framework for $\mathcal{M}$ theory, which is the theory which is conjectured to unify the different perturbative string theories. The mathematical structure should be closely related to the symmetries of string and $\mathcal{M}$ theory, but it should be unique, so that we are no longer allowed to ask, for example, why a theory based on $Osp(1|8)$ or $Osp(1|256)$ is not as good as one based on $Osp(1|32)$? The unique structure we seek should also be able to answer in a simple way some of the most puzzling aspects of string theory, such as why is it that the perturbative consistency of a quantum theory of gravity seems to require 6 more spatial dimensions than we observe, or why string theories require at least 16 supersymmetry generators, when none have so far been observed.

There is a very elegant answer to both of these questions which has been proposed from time to time[16, 18, 19], which is that the quantum geometry of space must involve the structures of the octonions. The idea, roughly, is that the local[1] structure of a $9+1$ dimensional spacetime may be expressed in terms of $J_2^{\mathcal{O}}$, which is the Jordan algebra of 2×2 hermitian matrices of octonions[2],

$$J_{\mathcal{O}}^2 = \begin{pmatrix} z_1 & \mathcal{O}_0 \\ \bar{\mathcal{O}}_0 & z_2 \end{pmatrix} \tag{1}$$

It is elementary to show that this is a representation of $9+1$ dimensional Minkowski spacetime, $M^{9,1}$. Here z_1 and z_2 can be taken as the light cone coordinates $z_\pm$ and the 8 transverse coordinates are coded in the octonion

[1] Because global symmetries can play no role in the formulation of a background independent gravitational theory, classical or quantum.

[2] For details about octonions and Jordan algebras, good references are [17, 18].

$\mathcal{O}_0$. Similarly, the relevant $Spin(9,1)$ spinors may be parameterized as 2 component octonionic spinors, which we may write as,

$$\Psi = \begin{pmatrix} \bar{\mathcal{O}}_2 \\ \mathcal{O}_1 \end{pmatrix} \tag{2}$$

Furthermore, several authors have pointed out that the structure of string theory and ten dimensional super-Yang-Mills theory requires identities that have to do with the existence of the octonions[16, 18, 19].

What would we observe if nature were like this? The problem is that it is hard to think of measurements that will produce octonionic valued quantities: most measurements give back only coarse grained, averaged quantities, which are usually rational numbers.

However, it is intriguing to wonder whether this may be the answer to the question we raised. One may conjecture that when octonionic variables are averaged over in some coarse graining procedure, they reduce to complex variables, as under the coarse graining the fields may forget the delicate algebraic relations that underlie the lack of associativity and commutivity of the octonions. Further, commutivity and associativity of measured values may be forced on us by the procedures by which we measure spacetime geometry. If this is the case then under coarse graining we would observe the fundamental structures reduced by a projection map[3]

$$\Pi : \mathcal{O} \rightarrow \mathbf{C}. \tag{3}$$

But under this map

$$J^2_{\mathcal{O}} \rightarrow J^2_{\mathbf{C}} \tag{4}$$

which is a representation of $3+1$ dimensional spacetime, while the spinors reduce to the two component chiral spinors of $SL(2,C)$.

The main idea of supersymmetry, in the context of a theory of spacetime, is that there should be some fundamental unification of the spacetime geometry with the fermionic degrees of freedom which live in the spinor representations of the local invariance group. There has recently been much speculation about the existence of an $\mathcal{M}$ theory, which unifies the different string theories in the context of an 11 dimensional structure. It is then very intriguing to notice that there is a way to unify the coordinates of the tangent space of a $9+1$ dimensional spacetime with its spinor degrees of freedom, in a way that naturally includes an eleventh spatial coordinate z_3.

[3]This is inspired by a conjecture of Dray and Magnogue[19].

This is to incorporate all of them in the algebra of 3×3 hermitian matrices of octonions, which is called the exceptional Jordan algebra. This is given by

$$J^3_{\mathcal{O}} = \begin{pmatrix} z_1 & \mathcal{O}_0 & \bar{\mathcal{O}}_2 \\ \bar{\mathcal{O}}_0 & z_2 & \mathcal{O}_1 \\ \mathcal{O}_2 & \bar{\mathcal{O}}_1 & z_0 \end{pmatrix} \tag{5}$$

We see that the three real and three octonionic variables gives us 27 degrees of freedom. In this letter we will show that these can parameterize a matrix model for string theory in which the vector and spinor of $9+1$ dimensional Minkowski spacetime are unified with an eleventh spatial coordinate. At the same time, the 27 degrees of freedom reminds us of a proposal to formulate $\mathcal{M}$ theory in terms of an extension of bosonic string theory to a 27 dimensional theory[30], as well as the idea that sixteen of the dimensions of bosonic string theory are transmuted from bosonic to fermionic by a dynamical mechanism that involves the decay of the tachyonic degree of freedom.

It then seems possible that the exceptional Jordan algebra is exactly the unique mathematical structure we seek. To investigate this hypothesis we here formulate a theory based on this algebra. We find that the exceptional Jordan algebra has invariants which allow the formulation of an apparently unique theory, when formulated in the language of the cubic matrix model[1, 2].

We may recall that the basic idea of the cubic matrix model is to build a background independent matrix model for string/M theory by starting with a matrix representation of Chern-Simons theory and then to extend it by taking degrees of freedom in an algebra associated with the conjectured tangent space symmetries relevant for string theory. By expressing the dynamics in terms of matrix Chern-Simons theory rather than a matrix model derived from a compactification of Yang-Mills theory, dependence on a particular background manifold and metric is avoided. At the same time, as shown in [2], this makes possible compactifications that give rise to completely manifold independent formulations of the theory[9, 5, 6] which are then seen to be described by a quantum deformation of the kinematical and dynamical structures of loop quantum gravity. In [1, 2] models were proposed based on $Osp(1|32)$ and its complexification $SU(16,16|1)$, here a simpler model of the same kind is proposed based on the exceptional Jordan algebra, $J^3_{\mathcal{O}}$. The $Osp(1|32)$ model was argued in [1] to reproduce both the dWHN-BFSS[20, 21, 22] and IKKT[23] models under particular compactifications, once the one loop corrections to the effective action were taken into account[4]. Here we show that a similar argument applied to $J^3_{\mathcal{O}}$ leads to a certain compactification of the matrix string theory described in [24, 25, 26] under which $SO(8)$ is broken to $G2$, the automorphism group of the octonions. We caution, however, that the argument here is based only on constraining the form of the effective action by its expected symmetries, detailed calculations based on a BRST quantization of Matrix Chern-Simons theory given in [27] are in progress and will be reported elsewhere.

Finally, we mention that B. Kim and A. Schwarz have proposed an elegant formulation of the IKKT model based on $J^2_{\mathcal{O}}$ and its spinor representation[28]. It is likely that that is related to a compactification of the model studied here.

2 The exceptional Jordan algebra

The exceptional Jordan algebra, J is composed of 3×3 hermitian matrices of octonions[17]. We will write the components as

$$J = \begin{pmatrix} z_1 & \mathcal{O}_0 & \bar{\mathcal{O}}_2 \\ \bar{\mathcal{O}}_0 & z_2 & \mathcal{O}_1 \\ \mathcal{O}_2 & \bar{\mathcal{O}}_1 & z_0 \end{pmatrix} \tag{6}$$

where $z_a \in R$ and $\mathcal{O}_a$ are octonions.

The automorphism group of J is known to be F_4. This group has several $Spin(8)$ subgroups, one of which acts on the components of J in the following way: the z_a are scalars, $\mathcal{O}_0, \mathcal{O}_1, \mathcal{O}_2$ are, respectively, the 8 dimensional vector, spinor and conjugate spinors. This is also an extension of this $Spin(8)$ subgroup to a $Spin(9)$ subgroup on which $(\mathcal{O}_1, \mathcal{O}_2)$ transforms as the 16 dimensional spinor representation.

We also note that there is an $SO(2)$ subgroup of F_4 such that $(z_1, z_2) = z_I$, with $I = 1, 2$ transform as the vector, while $(\mathcal{O}_1, \mathcal{O}_2)$ transforms as the spinor.

Among the automorphisms of J are a discrete set which we call the triality generators. This is an algebra generated by I, and ρ, where

$$\rho \circ J = \begin{pmatrix} z_2 & \mathcal{O}_1 & \bar{\mathcal{O}}_0 \\ \bar{\mathcal{O}}_1 & z_0 & \mathcal{O}_2 \\ \mathcal{O}_0 & \bar{\mathcal{O}}_2 & z_1 \end{pmatrix} \tag{7}$$

[4]A recent study of these models is reported in [3].

We note that $(\rho)^3 = I$. These operations are called triality because they mix up the vector, spinor and conjugate spinor representations of $SO(8)$, and hence generalize the duality that exchanges the two spinor representations of even spin groups.

In the following we will make use of the following properties of the algebra of octonions. An octonion will be written as $\mathcal{O} = o_{\vec{a}} e^{\vec{a}}$ with units $e^{\vec{a}}$, $\vec{a} = 0, 1, ..., 7 = (0, i)$. e^0 is the identity and the imaginary unitse^i, $i = 1, ..., 7$ satisfy

$$e^i e^j = -\delta^{ij} + \sigma^{ijk} e_k \tag{8}$$

where σ^{ijk} is completely antisymmetric and indices are raised and lowered with the flat metric δ_{ab} on R^8. The algebra is non-associative and the associator is defined as

$$(e^i e^j) e^k - e^i (e^j e^k) = \rho^{ijkl} e_l \tag{9}$$

ρ^{ijkl} is also completely antisymmetric and is equal to

$$\rho^{ijkl} = \frac{1}{3!} \epsilon^{ijklmno} \sigma_{mno} \tag{10}$$

The automorphism group of the algebra of octonions is $G2$, which is a 14 dimensional subalgebra of $Spin(7)$.

3 The model

We note that the $SO(8)$ representation content of J contains the fields of the matrix string model, plus an additional scalar degree of freedom. It is then very suggestive that this extra matrix degree of freedom, which is z_0, corresponds to the 11'th dimension of $\mathcal{M}$ theory. To test this idea we construct and study a matrix model based on J.

The degrees of freedom of our model will live in $\mathcal{G} \times J$, where $\mathcal{G}$ is a Lie algebra. In this letter we take $\mathcal{G} = U(P)$. A configuration of the system is then given by $J_I \otimes g^I$, where g^I are the generators of $\mathcal{G}$. We will sometimes write $J_A^B = J_I \otimes g^I{}_A{}^B$ where $g^I{}_A{}^B$ are the generators of $U(P)$ in the fundamental, P dimensional representation.

We want to make an action which is a functional of the $J_A{}^B$. If we follow the strategy of [1, 2] we may try to make a background independent model which should contain within it a topological quantum field theory. One way to do this is to construct a cubic action of the kind studied in [1, 2]. This means the action should be antisymmetrized in such a way that it reproduces the matrix form of Chern-Simons theory with the three $N \times N$ matrices z_a.

To construct the action we need a cubic product on J. There is a unique cubic product,

$$t(J_1, J_2, J_3) \rightarrow R \tag{11}$$

which is invariant under F_4. It is, however, completely symmetric.

The problem is that to get an action invariant under $\mathcal{G}$ we have to combine t with the structure constants of $\mathcal{G}$, f_{IJK}, which are antisymmetric. To combine them we need another antisymmetrization. One possibility is to use the triality map (7) to define an action:

$$S = \frac{k}{4\pi} f_{IJK} t(J^I, \rho \circ J^J, \rho^2 \circ J^K) \tag{12}$$

This action defines the theory, which we will call the *exceptional cubic matrix model.*

Expanded out in its $Spin(8)$ components, the action reads

$$S = \frac{k}{4\pi} f_{IJK} \epsilon^{abc} \left\{ 3x_a^I x_b^J x_c^K + 9x_c^K Re(\bar{\mathcal{O}}_a^I \mathcal{O}_b^J) - 3Re\left((\mathcal{O}_a^I \mathcal{O}_b^J) \mathcal{O}_c^K \right) \right\} + \sum_a f_{IJK} \sigma^{ijk} \mathcal{O}_{ai}^I \mathcal{O}_{aj}^J \mathcal{O}_{ak}^K \tag{13}$$

where $x_0 = z_1 + z_2$ and cyclic. We note that $Re[(O_1 O_2) O_3] = f(O_1, O_2, O_3)$, the triality map defined in [17] and $Re(\bar{O}_1 O_2) = (O_1, O_2)$ the inner product on the 8 dimensional space of octonions.

The first term will give rise to matrix chern-simons theory[1, 2]. This is the reason the coupling is written as $\frac{k}{4\pi}$. For the compactified action to be well defined under global gauge transformations, k must then be an integer.

The action has several symmetries. These include $Spin(8) \otimes U(P)$ transformations, triality and matrix translations,

$$J_A{}^B \rightarrow J_A{}^B + j\delta_A{}^B \tag{14}$$

where j is a member of the exceptional jordan algebra. We note that $Spin(8)$ acts differently on each of the three slots of the cubic product. The action is also invariant under $G2$ transformations which is the automorphism group which leaves the coefficients σ^{ijk} invariant. The three particular $Spin(8)$ subalgebras that act on each slot are related to each other by triality.

4 Three torus compactification

We now consider a standard matrix compactification[29, 1, 2] to a three-torus, on the three coordinates x_a. We break the $U(P)$ indices into indices $P, Q = 1, ..., N$ and i_0, i_1, i_2 such that $i_a = 1, ...M_a$ and $P = M_0 M_1 M_2 N$, We then write as usual

$$x_{aPi_a}^{Qj_a} = \partial_{a}{}_{i_a}^{j_a} I_P^Q + a_{ai_aP}^{j_aQ} \tag{15}$$

where the gauge fields $a_{ai_aP}^{j_aQ}$ are compactified as bosonic fields so that

$$a_{a(i_0+M_0i_1i_2)P}^{(j_0+M_0j_1j_2)Q} = a_{a(i_a)P}^{(j_a)Q} \tag{16}$$

We similarly compactify $\mathcal{O}_0$. However, $\mathcal{O}_1, \mathcal{O}_2$ are compactified as fermions, since they live in the spinor representations of $Spin(8) \oplus SO(2)$, so that

$$\mathcal{O}_{A(i+M_0)P}^{(j+M_0)Q} = -\mathcal{O}_{A(i)P}^{(j)Q} \tag{17}$$

for $SO(2)$ spinor indices $A = 1, 2$. We compactify with the same signs in the x^1 and x^2 directions.

The resulting action can be approximated by a continuum action for very large values of the M_a. We find

$$\begin{aligned} S &= \frac{k}{4\pi L_0 L_1 L_2} \oint_{T^3} d^3x Tr_N \left\{ [\epsilon^{abc}(a_a \partial_b a_c + \frac{2}{3} a_a a_b a_c) + \epsilon^{AB} Re(\bar{\mathcal{O}}_A \mathcal{D}_0 \mathcal{O}_B) \right\} \\ & \quad + f(\mathcal{O}_{0P}^{\ Q}, \mathcal{O}_{1Q}^{\ R}, \mathcal{O}_{2R}^{\ P}) + f_{IJK} \sigma^{ijk} \mathcal{O}_{0i}^I \mathcal{O}_{0j}^J \mathcal{O}_{0k}^K \end{aligned} \tag{18}$$

where $f(\mathcal{O}_0, \mathcal{O}_1, \mathcal{O}_2)$ is the triality map[17]. Here the compactification radii may be expressed as $L_a = l_{Planck} M_a$ where a dimensional scale is introduced for convenience and taken to be l_{Planck}. As in [1, 2] the physics is assumed to imply that the M_a are very big, but not infinite and that l_{Planck} denotes a fixed physical scale, which as in loop quantum gravity is related to minimal areas and volumes. Hence the model is finite and all quantum corrections are finite, although there may be infrared divergences as the M_a diverge.

We may note that there is a possible term which is linear in fermionic variables, of the form

$$(\mathcal{O}_{0P}^{\ Q}, \mathcal{D}_{1Q}^{\ R} \mathcal{O}_{2R}^{\ P}) - (\mathcal{O}_{0P}^{\ Q}, \mathcal{D}_{2Q}^{\ R} \mathcal{O}_{1R}^{\ P}). \tag{19}$$

However that term is inconsistent with the boundary conditions as a translation by M_0 takes it to minus itself, hence it vanishes and does not appear in the compactified action. Other terms of the form

$$\sum_{A=1}^{2} f_{IJK}\sigma^{ijk}\mathcal{O}^I_{Ai}\mathcal{O}^J_{Aj}\mathcal{O}^K_{Ak} \tag{20}$$

vanish once the spinorial variables $\mathcal{O}^J_{Aj}$ are taken to be fermions, by the combination of the three antisymmetries.

As a result, triality and $SO(2,1)$ are broken in the action, by the fermionic boundary conditions. But the action still has local $U(N)$ gauge symmetry, global $G2$ symmetry and matrix translation symmetry in the variables

$$a_{aP}^{\ Q}(x) \to a_{aP}^{\ Q}(x) + v_a\delta_P^{\ Q} \tag{21}$$

$$\mathcal{O}_{aP}^{\ Q}(x) \to \mathcal{O}_{aP}^{\ Q}(x) + o_a\delta_P^{\ Q} \tag{22}$$

where o_a is a triplet of octonions.

5 The one loop action and matrix string theory

The next step is computation of the effective action. While this has not yet been done we may use symmetries and power counting to constrain the form of the effective potential at one loop. At the one loop level we expect to have all terms of dimension 4 consistent with the symmetries. This gives us terms of the following forms (with $a = (0, x)$, and $x, y, = 1, 2$ the $2d$ spatial coordinates),

$$\begin{aligned} I^{\hbar} \;=\; & \frac{1}{L_0L_1L_2}\oint_{T^3} d^3xTr_N\{f_{xy}f^{xy} + f_{0x}f^{0x} + ([\mathcal{D}_x,\mathcal{O}_0],[\mathcal{D}^x,\mathcal{O}_0]) \\ & +([\mathcal{D}_0,\mathcal{O}_0],[\mathcal{D}^0,\mathcal{O}_0]) + \tau^{xAB}(\mathcal{O}_A,[\mathcal{D}_x,\mathcal{O}_B) + [\mathcal{O}_{0i},\mathcal{O}_{0j}][\mathcal{O}_{0k},\mathcal{O}_{0l}](\alpha\rho^{ijkl} + \beta\sigma^{ijn}\sigma^{kl}{}_n)\} \end{aligned} \tag{23}$$

where τ^{xAB} is the two dimensional Pauli matrix. We want to emphasize that from the symmetry analysis we cannot fix the coefficients, but only the terms.

To get to a theory related to the matrix string we take the limit $L_0 \to L_{Planck}$ so there is only the lowest mode in the 0 direction. We then drop terms in ∂_0. For consistency with the gauge invariance we must drop terms in $a_{0P}^{\ Q}$ at the same order.

The result is a two dimensional effective action which is of the form

$$\begin{aligned} I^{eff} \;=\; & I^0 + I^{\hbar} = \oint_{T^2} d^2xTr_N\{f_{xy}f^{xy} + (\mathcal{D}_x\mathcal{O}_0,\mathcal{D}^x\mathcal{O}_0) + \tau^{xAB}(\mathcal{O}_A,[\mathcal{D}_x,\mathcal{O}_B]) \\ & +f(\mathcal{O}_0,\mathcal{O}_1,\mathcal{O}_2) + [\mathcal{O}_{0i},\mathcal{O}_{0j}][\mathcal{O}_{0k},\mathcal{O}_{0l}](\alpha\rho^{ijkl} + \beta\sigma^{ijn}\sigma^{kl}{}_n)\} \end{aligned} \tag{24}$$

Using the triality, we may write the 8 dimensional vector representation as $\mathcal{O}_0 = V_{\vec{a}} e^{\vec{a}}$, where $\vec{a} = 0, ..., 7$ and $e^{\vec{a}}$ are the generators of the octonions. Given $Spin(8)$ spinor indices $\alpha = 1, ..., 8$ and $\bar{\alpha} = \bar{1}, ..., \bar{8}$ we then write $\mathcal{O}_1 = S_\alpha e^\alpha$ and $\mathcal{O}_2 = \bar{S}_{\bar{\alpha}} e^{\bar{\alpha}}$. We then have,

$$
\begin{aligned}
I^{eff} &= \oint_{T^2} d^2 x Tr_N \{ f_{12} f^{12} + (\mathcal{D}_I V_{\vec{a}} \mathcal{D}^I V^{\vec{a}} + \sigma^{IAB}(S_A, [\mathcal{D}_I, S_B]) + \Gamma^{\vec{a}\bar{\alpha}\alpha} V_{\vec{a}}, \bar{S}_{\bar{\alpha}} S_\alpha \\
&\quad + [V_i, V_j][V_k, V_l](\alpha \rho^{ijkl} + \beta \sigma^{ijn} \sigma^{kl}{}_n)\}
\end{aligned}
\tag{25}
$$

This is model has the degrees of freedom of the matrix string theory described in [26, 24, 25]. The only difference between this action and that of the matrix string is in the form of the four-matrix interaction terms among the $V_{\vec{a}}$, which reflect the fact that the $SO(8)$ symmetry of the bosonic fields of the matrix string has been broken to $G2$.

We may note that with particular coefficients on the cubic and quartic terms, the matrix string action is supersymmetric. It is interesting to conjecture that the supersymmetries are preserved under the breaking of $Spin(8)$ to $G2$ and that it can be understood as arising from the components of F_4 that live in the 8 dimensional spinor and conjugate spinor representations S and S_c. These act bosonically on the degrees of freedom in $J^3_{\mathcal{O}}$ however it is possible that after the compactifications that turn the $\mathcal{O}_A$ into fermions they may imply a fermionic symmetry of the reduced action. This is currently under investigation. In this regard we may note that manifolds with $G2$ holonomy have been studied before in connection with compactifications of supergravity and string theory[31].

6 Closing comments

In this note we have only introduced the cubic matrix model based on the exceptional Jordan algebra; many things need to be investigated. The first is to have some understanding of whether the quantization of the spinorial degrees of freedom as fermions is a free choice or is forced on us by something like a spin statistics theorem. Related to this is the question of whether the model is supersymmetric after the compactification proposed here, and whether the supersymmetries can be understood as related to that part of the F_4 algebra that is generated by $Spin(8)$ spinorial variables. It is also interesting to wonder whether the 27 components of $J^3_{\mathcal{O}}$ are the same 27 dimensions that have been recently proposed for a bosonic form of $\mathcal{M}$ theory[30]. Finally, it is very interesting to investigate whether compactifications such as described in [2] are allowed, as these will give a background independent phase which can be described in the language of quantum deformed loop quantum gravity, and thus show that there is a form of loop quantum gravity dual to the matrix string theory.

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